Splay trees

Deliverables

- Sequential Access Property
- Dynamic Finger Property
- Working Set Property
- Splay Tree definition
- Operations on splay trees

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Sequential Access Property

Sequential access is the property of a group of elements, that they must be accessed one after the other in a specific sequence.

Sequential Access Property: We can do inorder traversal of a BST in O(1) amortized time. Let the numbers of the set {1,2,...n} be keys stored in a binary search tree. Let $\{x_1,x_2,...x_m\}$ be keys of an access pattern x_i.

It is known that some access patterns are very easy . For example if $x_i=k$ for all i, then placing k at the root will be O(1) access time. Similarly inorder access pattern will also give us constant amortized time.

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Static Optimality of BST

let k be a key which appears in access sequence with probability p_k. Then per operation cost of access is $O(\sum p_i(\lg(1/p_i))$

This is because an element which appears with probability p_i can be stored at height $\lg p_i$. This is strict lower bound for static sets.

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Dynamic Finger Property

Dynamic finger property : if last access was to key x_{i-1} and the present access is to key x_i then the cost of the present access is $O(\lg|x_i-(x_{i-1})|)$

This property performs well on access patterns which exhibit spatial locality.

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Working Set Property

Working Set property: Let t_j the number of distinct keys between the present operation and the last time the same key was accessed. Let t_j be the number of distinct keys in $\{x_i..x_j\}$ then the cost of accessing x_j is $O(\lg t_j+2)$

Data structures with this property perform well on access patterns which exhibit temporal locality.

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Unified Property

Unified Property: Let t_{ij} be the number of distinct keys in $\{x_i \ldots x_j\}$ then the cost of accessing x_j is $O(\lg \min_i (|x_i - x_j| + t_{ij} + 2)$

It says that access to keys close to keys which have been recently accessed is cheap.

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Splay trees

Splay Trees are Binary Search Trees

Searching in a Splay Tree: Starts the Same as in a BST

Splay trees are much simpler then AVL or Red Black trees. Their average performance is comparable and they provide additional properties not found in AVL or RB trees. It has working set and dynamic finger property.

Splay tree is a binary search tree where a node is splayed after it is accessed (for a search or update)

Deepest internal node accessed is splayed

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Splay trees-complexity

- splaying costs O(h), where h is height of the tree – which is still O(n) worst-case
- O(h) rotations, each of which is O(1)
- Thus, amortized cost of any splay operation is O(log n).
- In fact, the analysis goes through for any reasonable definition of rank(x).
- Thus splay trees can adapt to perform searches on frequently-requested items much faster than O(log n).

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Splay algorithm

All normal operations on a binary search tree are combined with one basic operation, called splaying. Splaying the tree for a certain element rearranges the tree so that the element is placed at the root of the tree.

One way to do this is to first perform a standard binary tree search for the element in question, and then use tree rotations in a specific fashion to bring the element to the top.

Alternatively, a top-down algorithm can combine the search and the tree reorganization into a single phase

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Which nodes are splayed

findElement	if key found, use that node if key not found, use parent of ending external node
insertElement	use the new node containing the item inserted
removeElement	use the parent of the internal node that was actually removed from the tree (the parent of the node that the removed item was swapped with)

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Splaying

When a node x is accessed, a splay operation is performed on x to move it to the root. To perform a splay operation we carry out a sequence of splay steps, each of which moves x closer to the root. By performing a splay operation on the node of interest after every access, the recently accessed nodes are kept near the root and the tree remains roughly balanced, so that we achieve the desired amortized time bounds.

Each particular step depends on three factors:

Whether x is the left or right child of its parent node, y,

whether y is the root or not, and if not

whether y is the left or right child of its parent, g (the grandparent of x).

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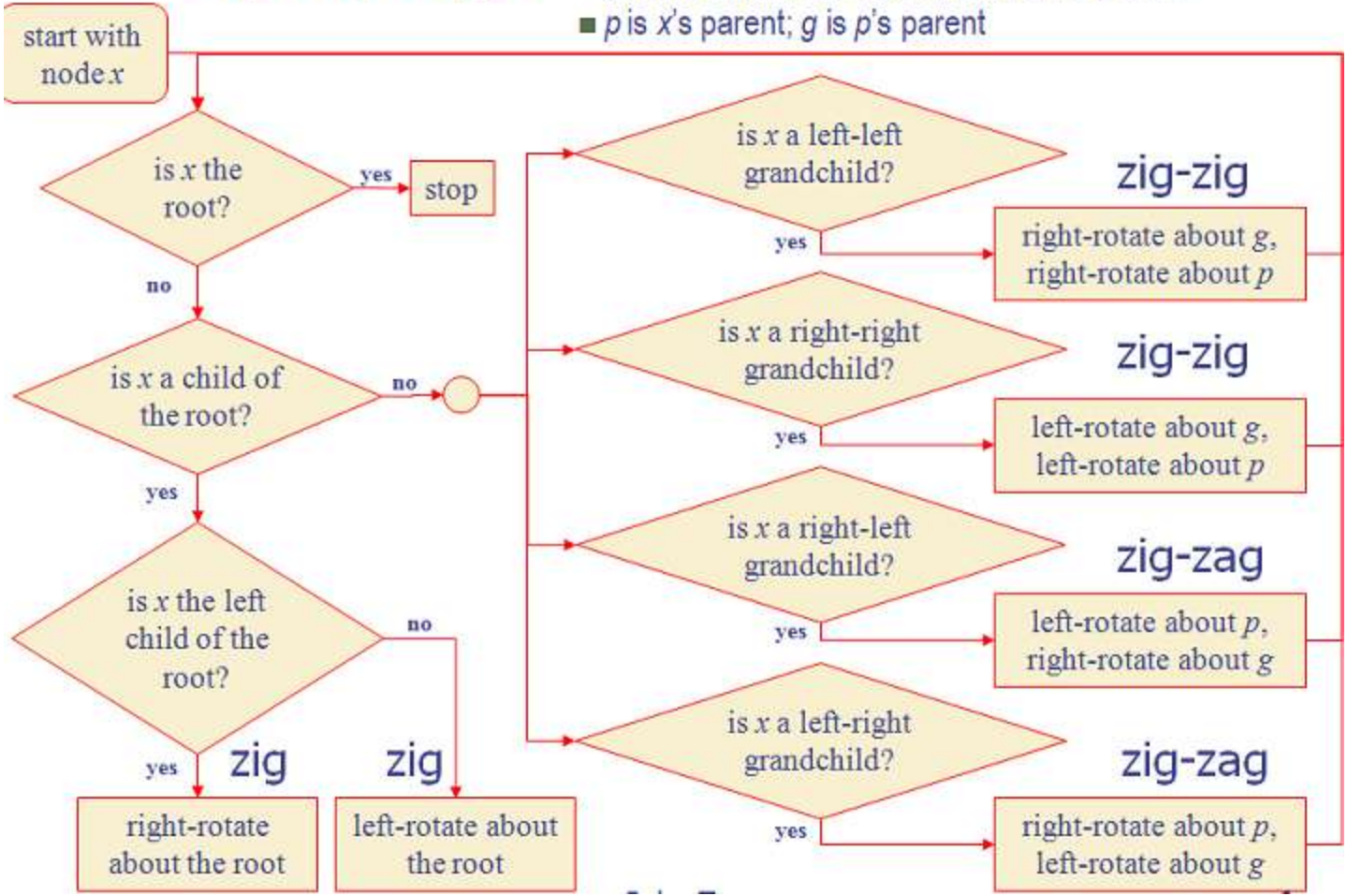
Splaying:

- "x is a left-left grandchild" means x is a left child of its parent, which is itself a left child of its parent
- p is x's parent; g is p's parent



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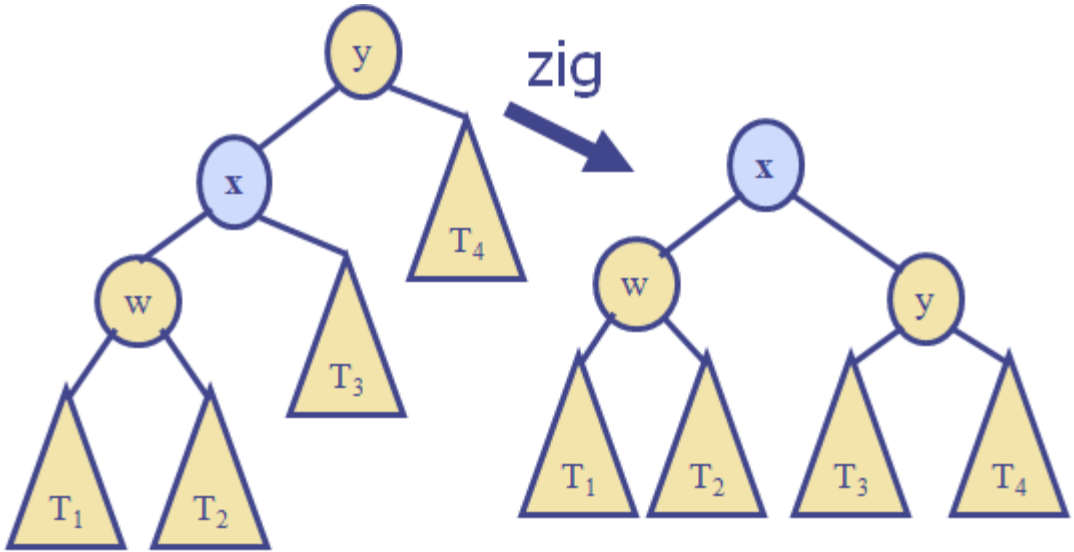
Zig



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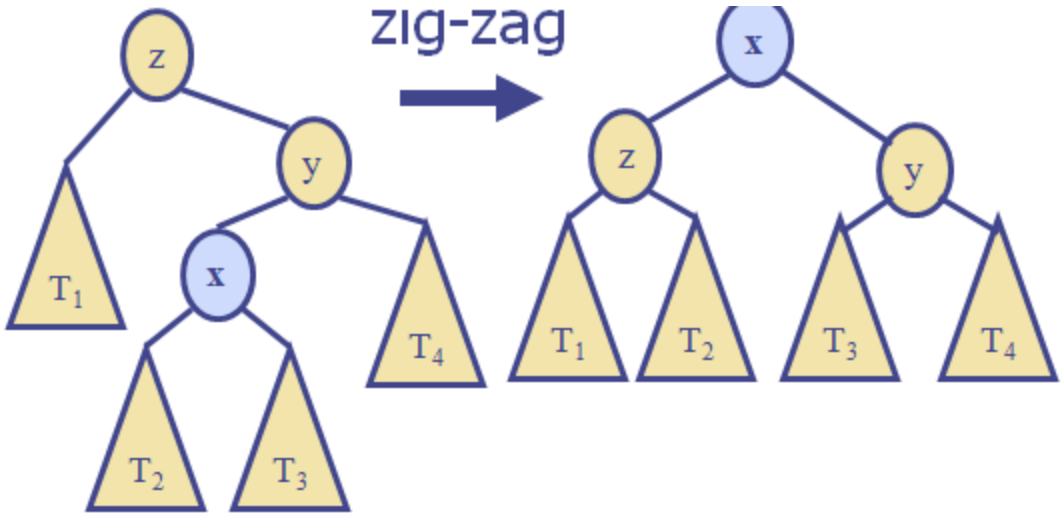
Zig-Zag



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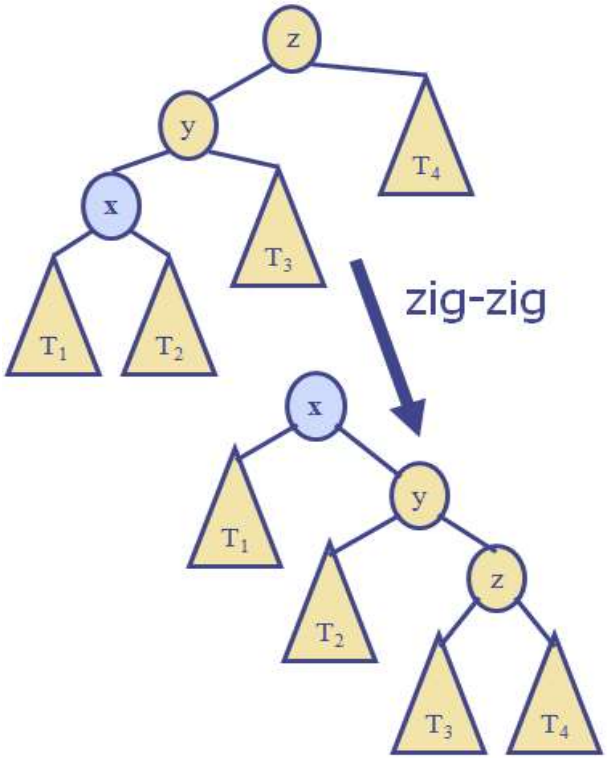
Zig-Zig



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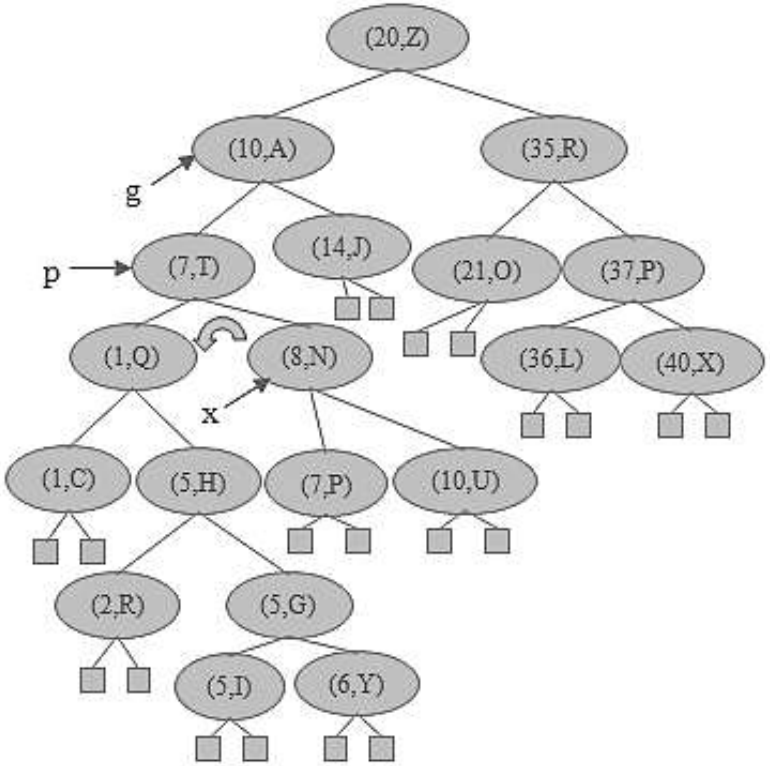
X=(8,N) Before rotating

x is the right child of its parent, which is the left child of the grandparent left-rotate around *p*, then right-rotate around *g*



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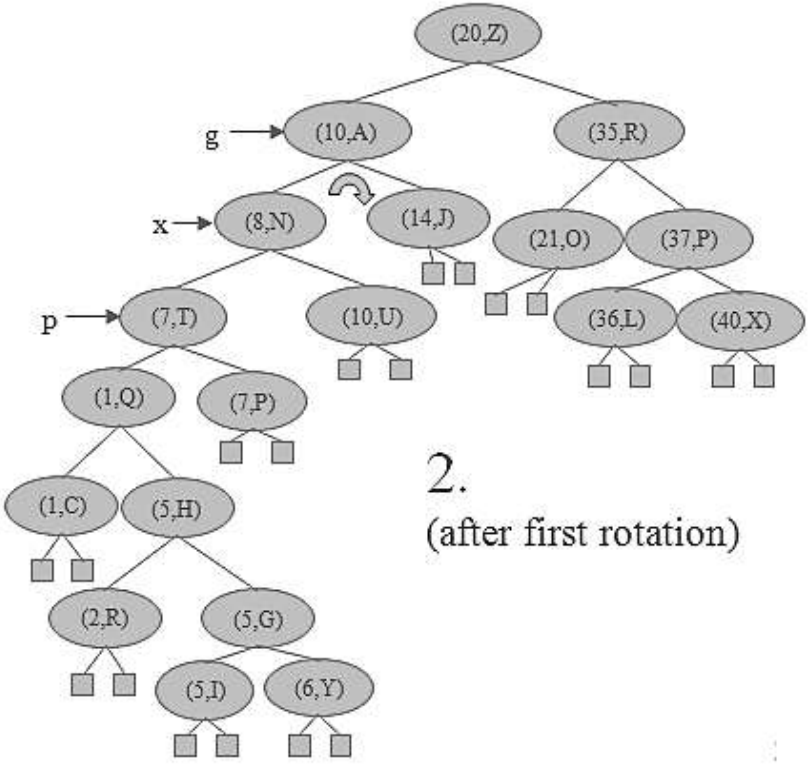
After First Rotation



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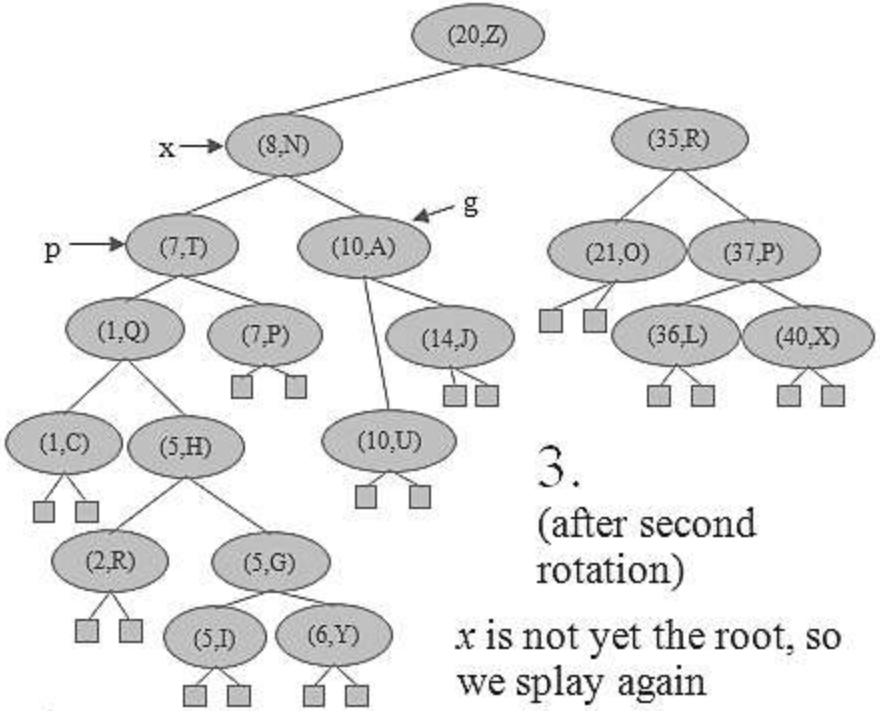
After Second Rotation



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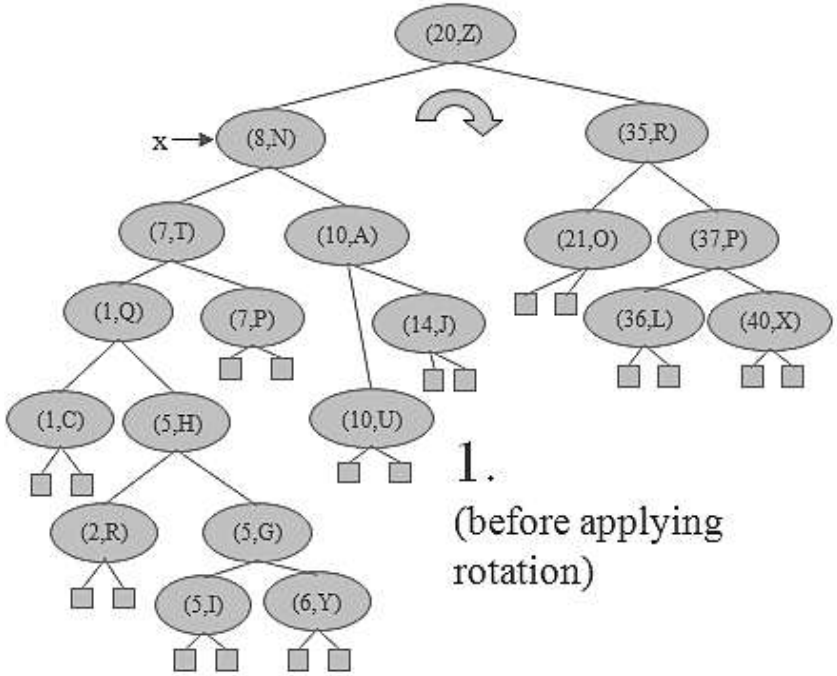
Before Rotation

Now x is the left child of the root right-rotate around root.



1.
(before applying rotation)

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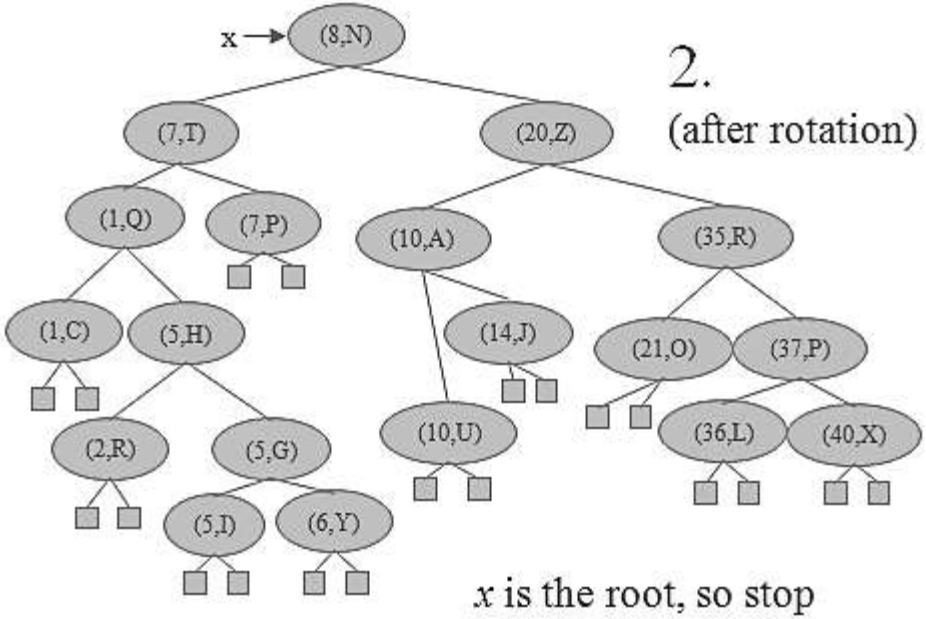
Last step



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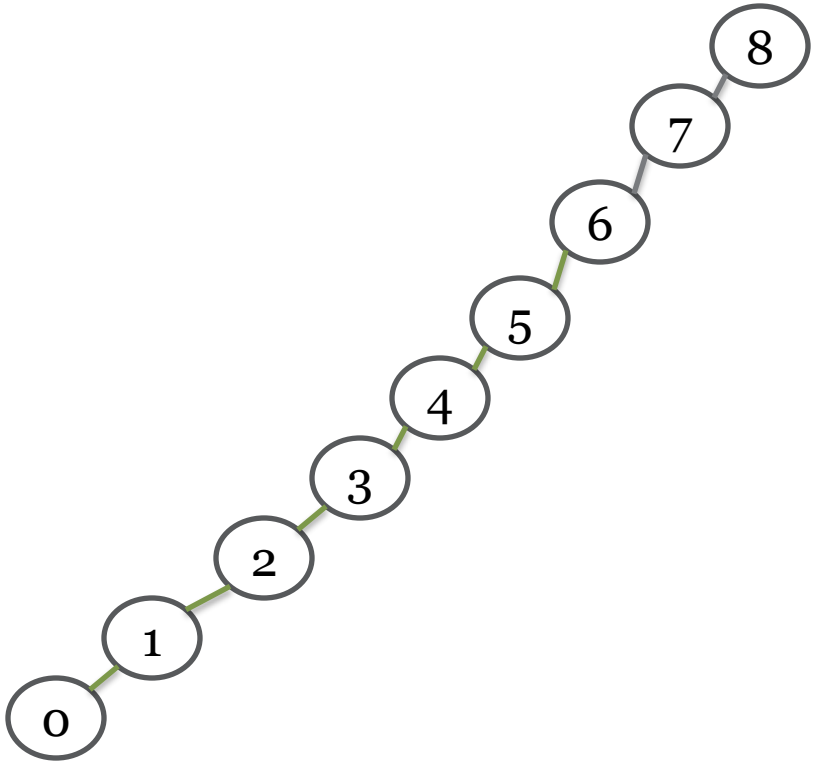
Example Insert 0,1,..8



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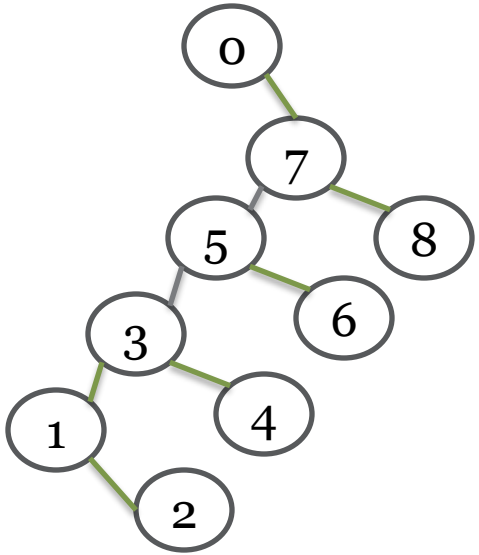
After find 0



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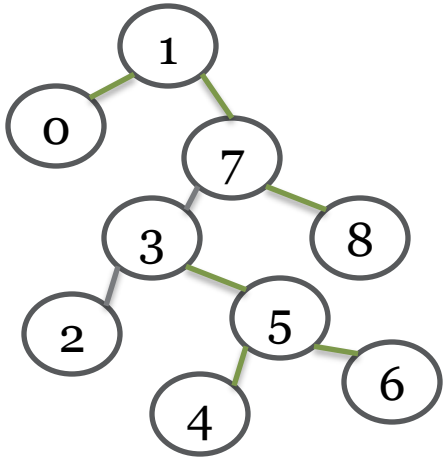
After find 1



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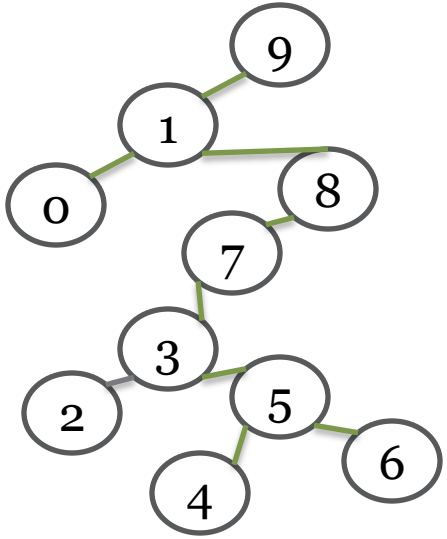
After Insert 9



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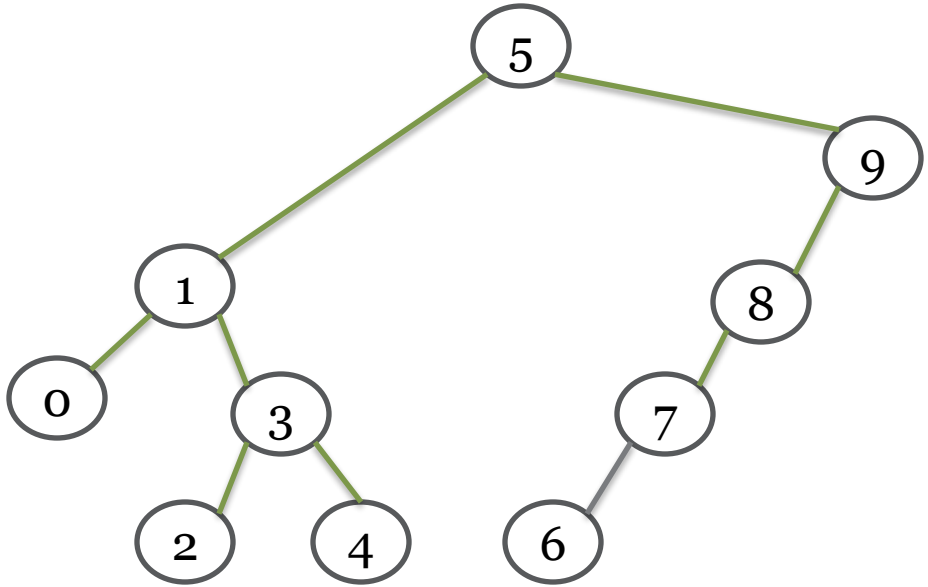
Deleting Node 5 Searching 5



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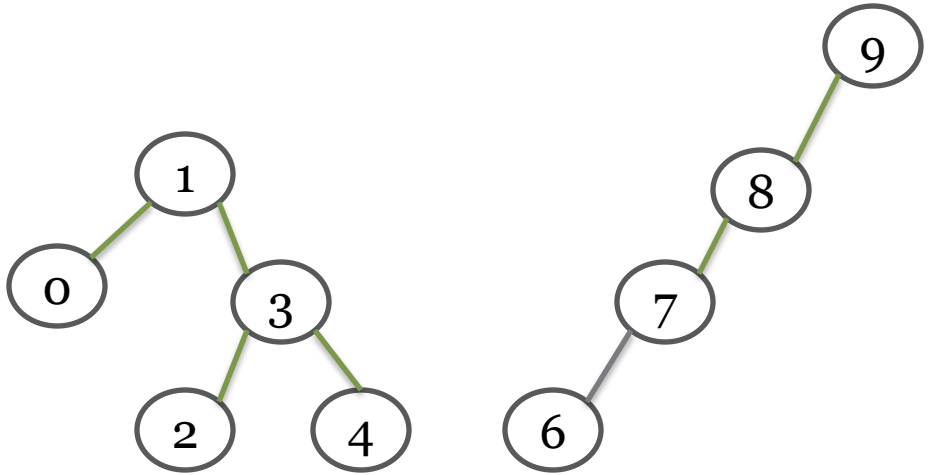
Deleting Node 5 Removing Root



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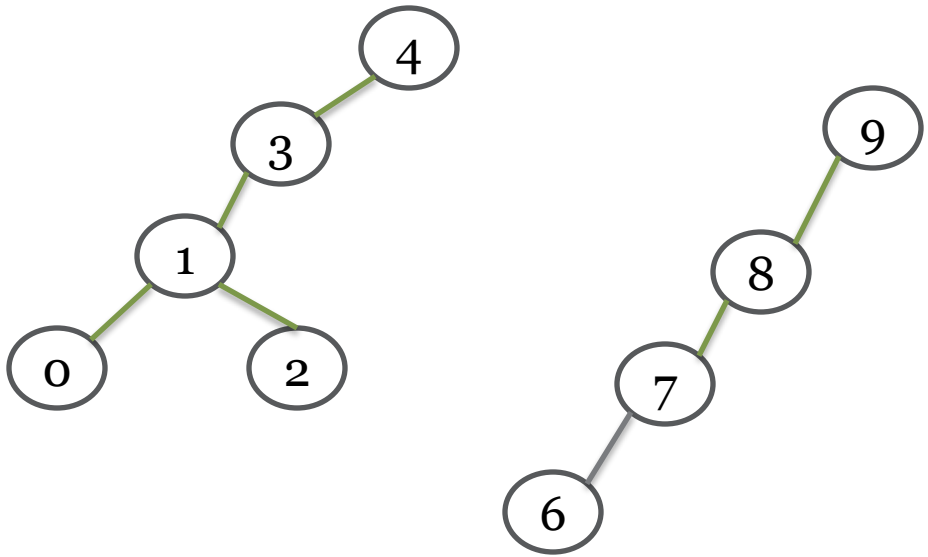
Deleting Node 5 Searching Largest



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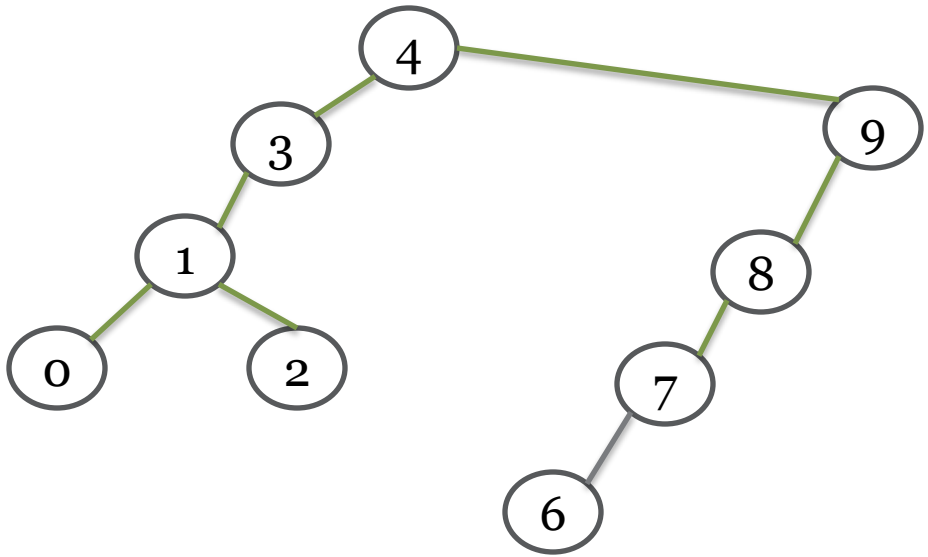
Deleting Node 5 Linking new root



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Questions, Comments and Suggestions

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Question 1

Splaying the tree for a certain element rearranges the tree so that the element is placed at the ___of the tree.

A) Leaves

B) Root

C) Top Half

D) Bottom Half

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Question 2

Amortized or Randomized access cost of splay tree is

A) O(n)

B) O(nlogn)

C) O(logn)

D) $O(n^2)$

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Question 3

A splay tree can be worse than a static tree in terms of any operation by a

A) logn factor

B) n factor

C) Constant Factor

D) lg^2n

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