

An Approach to Efficient Network Flow Algorithm for Solving Maximum Flow Problem

**Thesis submitted in partial fulfillment of the requirements for the award
of degree of**

**Master of Engineering
in
Computer Science & Engineering**

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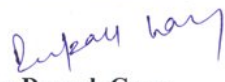
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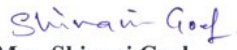
I hereby certify that the work which is being presented in the thesis entitled, “**An approach to efficient Network Flow Algorithm for solving Maximum Flow Problem**”, in partial fulfillment of the requirements for the award of degree of Master of Engineering in Computer Science and Engineering submitted in Computer Science and Engineering Department of Thapar University, Patiala, is an authentic record of my own work carried out under the supervision of *Dr. Deepak Garg & Mrs. Shivani Goel* and refers other researcher’s works which are duly listed in the reference section.

The matter presented in this thesis has not been submitted for the award of any other degree of this or any other university.



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This is to certify that the above statement made by the candidate is correct and true to the best of my knowledge.


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Abstract

Network Flow Problems have always been among the best studied combinatorial optimization problems. These problems are central problems in operations research, computer science, and engineering and they arise in many real world applications. Flow networks are very useful to model real world problems like, current flowing through electrical networks, commodity flowing through pipes and so on. Maximum flow problem is the classical network flow problem. In this problem, the maximum flow which can be moved from the source to the sink is calculated without exceeding the maximum capacity. Once, the maximum flow problem is solved it can be used to solve other network flow problems also. Maximum flow problem is thoroughly studied in this thesis and the general algorithm is explained in detail to solve it. Then other network flow problems like, Minimum Cost Flow, Transshipment, Transportation, and Assignment problems are also briefly explained and shown that how they can be converted into maximum flow problem.

The Ford-Fulkerson algorithm is the general algorithm which can solve all the network flow problems. The improvement of the Ford Fulkerson algorithm is Edmonds-Karp algorithm which uses BFS procedure instead of DFS to find an augmenting path.

Next the modified Edmonds-Karp algorithm is designed to solve the maximum flow problem in efficient manner. One real world problem is taken, it is converted into network flow graph and the new algorithm is implemented to solve the problem. The same problem is solved using Edmonds-Karp algorithm also and both algorithms are compared in terms of different parameters. Finally, it is proved that the modified algorithm performs better in most cases and the new algorithm is implemented in C.

Table of Contents

Certificate	i
Acknowledgement	ii
Abstract	iii
Table of Contents	iv
List of Figures	vi
List of Tables	vii
Chapter 1 Introduction	1
1.1 Introduction to Network Flow Problems	2
1.2 Relationship between Network Flow Problems.....	3
Chapter 2 Literature Review	5
2.1 Introduction to Network Flow Concepts.....	5
2.2 Networks with Multiple Sources and Sinks.....	7
2.3 Maximum Flow Problem	8
2.3.1 The Ford-Fulkerson Method	9
2.3.2 Residual Networks.....	10
2.3.3 Augmenting Paths.....	12
2.3.4 Cuts of Flow Networks	12
2.3.5 The Basic Ford-Fulkerson Algorithm.....	14
2.3.5.1 Analysis of Ford-Fulkerson	16
2.3.5.2 Limitation of Ford-Fulkerson Algorithm.....	17
2.3.6 The Edmonds-Karp Algorithm	17
2.4 Bipartite Matching	18
2.4.1 The Maximum Bipartite Matching Problem.....	18
2.4.2 Finding Maximum Bipartite Matching	19
2.5 Minimum Cost Flow	20

2.6 Transshipment Problem	23
2.7 Transportation Problem	25
2.8 The Assignment Problem	26
Chapter 3 Problem Statement	29
3.1 Methodology.....	29
Chapter 4 Design and Implementation	30
4.1 Converting the Problem into Graph.....	30
4.2 Solution using Modified Edmonds-Karp Algorithm	32
4.3 Solution using Edmonds-Karp Algorithm	37
4.4 Discussion and Results.....	38
4.5 Results	40
4.6 Implementation	41
Chapter 5 Conclusion and Future Scope	42
5.1 Future Scope	42
References	43
List of Publications	45
Appendix A	46
A.1 Implementation Code.....	46
A.2 Input File	49
A.3 Output File	50

List of Figures

Figure 1.1 Relationship between network flow problems	4
Figure 2.1 Sample flow network graph	5
Figure 2.2 Maximum-flow problem with multiple-source, multiple-sink	8
Figure 2.3(a)-(d) The flow network showing augmenting path and the flow	11
Figure 2.4 A cut in the flow network.....	13
Figure 2.5(a)-(d) The execution of the basic Ford-Fulkerson algorithm.....	15
Figure 2.6 A flow network showing limitation of the Ford-Fulkerson algorithm.....	17
Figure 2.7 A bipartite graph $G = (V, E)$ with vertex partition $V = L \cup R$	19
Figure 2.8 The flow network corresponding to a bipartite graph.....	20
Figure 2.9 Side-by-side computation showing difference when considering the minimum cost flow.....	22
Figure 2.10 Sample Transshipment problem instance converted to Minimum Cost Flow Problem instance.....	24
Figure 2.11 A simple transportation model.....	26
Figure 2.12 The assignment problem cast as a transportation network	27
Figure 4.1 The initial flow network corresponding to the problem to be solved	31
Figure 4.2 Residual Graph after 0 augmentation & flow graph after 1 st augmentation	33
Figure 4.3 Residual Graph after 1 st augmentation & flow graph after 2 nd augmentation .	33
Figure 4.4 Residual Graph after 2 nd augmentation & flow graph after 3 rd augmentation .	34

Figure 4.5 Residual Graph after 3 rd augmentation & flow graph after 4 th augmentation ..	34
Figure 4.6 Residual Graph after 4 th augmentation & flow graph after 5 th augmentation .	35
Figure 4.7 Residual Graph after 5 th augmentation & flow graph after 6 th augmentation .	36
Figure 4.8 Residual Graph after 6 th augmentation	36

List of Tables

Table 2.1 Person-task relationship for assignment problem	27
Table 4.1 Defined capacities of each pipeline between two areas	30
Table 4.2 Correspondence between areas and vertices	31
Table 4.3 Iteration – augmentation comparison of two algorithms for the problem	38
Table 4.4 Comparison of complexities of two algorithms	39